ON THE FUNDAMENTAL GROUPS OF THE COMPLEMENTS OF HURWITZ CURVES

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Abstract. It is proved that the commutator subgroup of the fundamental group of the complement of any plane affine irreducible Hurwitz curve (respectively, any plane affine irreducible pseudo-holomorphic curve) is finitely presented. It is shown that there exists a pseudo-holomorphic curve (a Hurwitz curve) in $\mathbb{CP}^2$ whose fundamental group of the complement is not Hopfian and, respectively, this group is not residually finite. In addition, it is proved that there exist an irreducible nonsingular algebraic curve $C \subset \mathbb{C}^2$ and a bi-disk $D \subset \mathbb{C}^2$ such that the fundamental group $\pi_1(D \setminus C)$ is not Hopfian.

0. Introduction

The notion of Hurwitz curves with respect to a linear projection of the projective plane $\mathbb{CP}^2$ to $\mathbb{CP}^1$ was introduced in [9] and is a natural generalization of the notion of the plane algebraic curves (in [9], Hurwitz curves are called "semi-algebraic curves"). A precise definition of Hurwitz curves can be found, for example, in [3]. Roughly speaking, Hurwitz curves in $\mathbb{CP}^2$ imitate the behavior of plane algebraic curves with respect to the pencil of complex lines defining the projection. In particular, they look like analytic curves in neighborhoods of critical points of the projection.

Hurwitz curves play an important role in symplectic geometry. In particular, Auroux and Katzarkov (see [1], [2]) proved that a compact symplectic 4-manifold (X, ω) with symplectic form ω, whose class $[\omega] \in H^2(X, \mathbb{Z})$ and for which an ω-compatible almost complex structure J is chosen, can be presented as an approximately holomorphic generic covering $f_k : X \to \mathbb{CP}^2$, $k >> 0$, branched over a cuspidal Hurwitz curve $\bar{H}_k$ (maybe with negative nodes), where f_k is given by three sections of the line bundle $L^{\otimes k}$ and L is a line bundle on X whose first Chern class is $[\omega]$. Therefore the investigation of the fundamental groups $\pi_1(\mathbb{CP}^2 \setminus \bar{H})$ of the complements of Hurwitz curves $\bar{H}$ is very important for symplectic geometry.

In [4], a class $\mathcal{C}$ of groups, called C-groups, was defined. By definition, this class consists of the groups G which are given by finite presentations of the following form: for some integer m, a subset

$$J = \{\, (i,j,k) \in \mathbb{Z}^3 \mid 1 \leqslant i,j \leqslant m,\ 1 \leqslant k \leqslant h(i,j)\},$$

where $h : \{1,\dots,m\}^2 \to \mathbb{Z}$ is a function, and a subset $W = \{w_{i,j,k} \in \mathbb{F}_m \mid (i,j,k) \in J\}$ of words in a free group $\mathbb{F}_m$ generated by an alphabet $\{x_1,\dots,x_m\}$ (it is possible that $w_{i_1,j_1,k_1} = w_{i_2,j_2,k_2}$ for $(i_1,j_1,k_1) \neq (i_2,j_2,k_2)$), a group $G \in \mathcal{C}$ possesses the presentation

$$G_W =< x_1,\dots,x_m \mid x_i = w^{-1}_{i,j,k} x_j w_{i,j,k},\ w_{i,j,k} \in W > . \qquad (1)$$

A C-group G is called *irreducible* if $G/G' \simeq \mathbb{Z}$, where $G' = [G,G]$ is the commutator subgroup.

Denote by $\varphi_W : \mathbb{F}_m \to G_W$ the canonical epimorphism. The elements $\varphi_W(x_i) \in G_W$, $1 \leqslant i \leqslant m$, and the elements conjugated to them are called the *C-generators* of the C-group $G = G_W$. Let $f : G_1 \to G_2$ be a homomorphism of C-groups. It is called a *C-homomorphism* if the images of the C-generators of G_1 under f are C-generators of the C-group G_2. We will distinguish C-groups up to C-isomorphisms.

Note that the class $\mathcal{C}$ contains the subclasses $\mathcal{K}$ and $\mathcal{L}$, respectively, of the knot and link groups given by Wirtinger presentation. In [4], it was proved that the class $\mathcal{C}$ coincides with the class of the fundamental groups of the complements of orientable closed surfaces in the 4-dimensional sphere S^4 (with generalized Wirtinger presentation). Besides, it follows from Theorem 1.14 in [6] and Theorem 2.1 in [3] that for any C-group G there are an affine nonsingular algebraic curve $C \subset \mathbb{C}^2$ and a bi-disk $D = \{\, (z,w) \in \mathbb{C}^2 \ \mid \mid z \mid \leqslant 1,\ \mid w \mid \leqslant 1\}$ such that $G \simeq \pi_1(D \setminus C)$.

Let $H \subset \mathbb{C}^2 = \mathbb{CP}^2 \setminus L_\infty$ be an affine Hurwitz curve of degree m, that is, $H = \bar{H} \cap \mathbb{C}^2$, where $\bar{H}$ is a Hurwitz curve in $\mathbb{CP}^2$ with respect to some pencil of lines and a line L_∞ is a member of the pencil being in general position with respect to $\bar{H}$, and the intersection number $\bar{H} \cdot L_\infty = m$. Denote by $\mathcal{H} = \{\, \pi_1(\mathbb{C}^2 \setminus H)\,\}$ the class of the fundamental groups of the complements of the affine Hurwitz curves H. If $\bar{H}$ is a Hurwitz curve of degree m, then the Zariski – van Kampen presentation of $\pi_1(\mathbb{C}^2 \setminus H)$ (defined by the pencil of lines) is a presentation of the form (1) such that the words $w_{i,i,1} = x_1 \dots x_m$, $i = 1,\dots,m$, belong to W, i.e., the element $x_1 \dots x_m$ belongs to the center of G_W. In [6], it was proved that if a C-group G_W is given by presentation (1) such that the element $x_1 \dots x_m$ belongs to the center of G_W, then for some $k \in \mathbb{N}$ there is a Hurwitz curve $\bar{H} \subset \mathbb{CP}^2$ of degree $M = m2^k$ such that $G_W = \pi_1(\mathbb{C}^2 \setminus H)$ and the element $(x_1 \dots x_m)^{2^k} \in G_W$ corresponds

to a circuit around L_∞. One can add the generators $x_{m+1},\dots,x_{2^km}$ and relations $x_i=x_{i+m}$ for $i=1,\dots,(2^k-1)m$ to presentation (1) of G_W and obtain a group isomorphic to G_W. Note that relations $x_i=x_j$ are C-relations, since they can be written as $x_j=x_j^{-1}x_ix_j$. Therefore the class $\mathcal{H}$ coincides with the subclass of $\mathcal{C}$ consisting of the groups G which possess presentations (1) for some m and such that the elements $x_1\dots x_m$ (for some m for each group $G\in\mathcal{H}$) belong to the centers of these groups, since we consider C-groups up to C-isomorphisms. A group $G\in\mathcal{H}$ will be called *a Hurwitz C-group of degree m* if C-generators $x_1,\dots,x_m$ of G generate the group G and the element $x_1\dots x_m$ belongs to the center of G (note that the degree of a Hurwitz C-group G is not defined canonically and depends on its C-presentation).

As is known the commutator subgroups G' of a lot of irreducible C-groups G are not finitely generated (in particular, G' of $G\in\mathcal{K}$ is finitely generated group iff G is the group of a fibred knot (see [10]), and, moreover, in the case of a fibred knot G' is a free group). One of the main results of this article is the following theorem.

Theorem 0.1. *Let $G=\pi_1(\mathbb{C}^2\setminus H)\in\mathcal{H}$, where $\bar{H}$ is an irreducible Hurwitz curve. Then the commutator subgroup $G'=[G,G]$ is a finitely presented group.*

Note that Theorem 0.1 is a generalization of the similar result in algebraic case (see [5]).

For any C-group $G=G_W$ with presentation (1), denote by $\nu: G\to\mathbb{F}_1$ the natural C-homomorphism and $N=\ker\nu$. Since $N=[G,G]$ for an irreducible C-group G, Theorem 0.1 is a simple consequence of the following

Theorem 0.2. *For any Hurwitz C-group $G\in\mathcal{H}$ the group N is finitely presented.*

In difference to [5], the proof of Theorem 0.2 given given in section 1 is purely algebraic.

Let J be an almost complex structure in $\mathbb{CP}^2$ compactible with Fubini – Studi symplectic form and $\bar{H}$ be a J-holomorphic curve in $\mathbb{CP}^2$. If we chose a pencil of pseudo-holomorphic lines, then, by Zariski – van Kampen Theorem, a presentation of $\pi_1(\mathbb{CP}^2\setminus(\bar{H}\cup L_\infty))$ is defined by braid monodromy factorization of $\bar{H}$ with respect to the chosen pencil, where L_∞ is one of the J-lines of the pencil being in general position with respect to $\bar{H}$. Therefore similar to the case of Hurwitz curves, it is easy to show (see the proof of Theorem 6.1 in [6]) that $\pi_1(\mathbb{CP}^2\setminus(\bar{H}\cup L_\infty))$ is a Hurwitz C-group. Thus, we have

Corollary 0.3. *Let $\bar{H}$ be an irreducible pseudoholomorphic curve in $\mathbb{CP}^2$. Then the commutator subgroup G' of $G = \pi_1(\mathbb{CP}^2 \setminus (\bar{H} \cup L_\infty))$ is a finitely presented group.*

Let $\bar{H}$ be a Hurwitz curve of degree m. To obtain a presentation of $\pi_1(\mathbb{CP}^2 \setminus \bar{H})$ from Zariski – van Kampen presentation (1) of the group $\pi_1(\mathbb{C}^2 \setminus H)$, it is sufficient to add the additional relation $x_1 \dots x_m = 1$ (the element $x_1 \dots x_m \in \pi_1(\mathbb{C}^2 \setminus H)$ corresponds to a circuit around the line L_∞).

For any Hurwitz C-group $G = G_W$ of degree m given by presentation (1), denote by

$$\bar{G}_{m,k} = G_W / \{(x_1 \dots x_m)^k = 1\}$$

and call $\bar{G}_{m,k}$ a *projective Hurwitz group of degree mk*. It is easy to see that the homomorphism ν induces the homomorphism $\nu_{mk} : \bar{G}_W \to \mathbb{Z}/mk\mathbb{Z}$. Put $\bar{N}_{m,k} = \ker \nu_{mk}$. The following theorem is a particular case of Corollary 2.8. in [8].

Theorem 0.4. *For any projective Hurwitz group $\bar{G}_{m,k}$ of degree mk the group $\bar{N}_{m,k}$ is finitely presented.*

Since for an irreducible Hurwitz curve (respectively, for an irreducible pseudo-holomorphic curve) $\bar{H}$ of $\deg \bar{H} = m$ the commutator subgroup G' of $G = \pi_1(\mathbb{CP}^2 \setminus \bar{H})$, given by Zariski – van Kampen presentation, coincides with $\bar{N}_{m,1}$, we have

Corollary 0.5. *Let $\bar{H} \subset \mathbb{CP}^2$ be an irreducible Hurwitz curve (respectively, pseudo-holomorphic curve). Then the commutator subgroup G' of $G = \pi_1(\mathbb{CP}^2 \setminus \bar{H})$ is a finitely presented group.*

Let C be a plane algebraic curve. In [11], O. Zariski formulated the following question:

Is $G = \pi_1(\mathbb{CP}^2 \setminus C)$ a residually finite group?

It is natural to ask the same question in the local case, i.e., if $G = \pi_1(D \setminus C)$, where D is a bi-disk in $\mathbb{C}^2$, and in the cases of Hurwitz C-groups and projective Hurwitz groups.

Theorem 0.6. *There are*

(*i*) *an irreducible C-group $\tilde{G}$,*

(*ii*) *a Hurwitz C-group G of degree three for which $G/G' \simeq \mathbb{Z}^2$*

such that $\tilde{G}$, G, and the projective Hurwitz groups $\bar{G}_{3,k}$, $k \in \mathbb{N}$, associated with G, are non Hopfian. In particular, they are not residually finite groups.

Applying Theorems 1.14, 6.2 in [6] and Theorem 2.1 in [3], we obtain

Corollary 0.7. *There are*

(*i*) *an irreducible nonsingular algebraic curve* $C \subset \mathbb{C}^2$ *and a bi-disk* $D = \{ (z, w) \in \mathbb{C}^2 \mid |z| \leqslant 1, \ |w| \leqslant 1 \}$;

(*ii*) *a Hurwitz curve* $\bar{H} \subset \mathbb{CP}^2$ *consisting of two irreducible components*

whose groups $\pi_1(D \setminus C)$, $\pi_1(\mathbb{CP}^2 \setminus \bar{H})$, *and* $\pi_1(\mathbb{C}^2 \setminus H)$ *are non-Hopfian and, in particular, they are not residually finite groups.*

Note that, contrary to Corollary 0.7 (i), if $C \subset \mathbb{C}^2$ is a non-singular algebraic curve meeting transversally the line at infinity, then, by Zariski Theorem, $\pi_1(\mathbb{C}^2 \setminus C)$ is an abelian group. Note also that it follows from the proof of Theorem 6.2 in [6] that the Hurwitz curve $\bar{H}$ in Corollary 0.7 (ii) can be chosen such that all its singular points are simple triple points, i.e., locally they can be given by equation $w(w^2 - z^2) = 0$. Applying rescaling, one can assume that $\bar{H}$ is a symplectic surface. Therefore we have

Corollary 0.8. *There exists a pseudo-holomorphic curve* $\bar{H} \subset \mathbb{CP}^2$ *consisting of two irreducible components, having simple triple points as its singularities and such that* $\pi_1(\mathbb{CP}^2 \setminus \bar{H})$ *is not Hopfian.*

The proof of Theorem 0.6 is given in section 2.

1. Proof of Theorem 0.2

Obviously, Theorem 0.2 follows from

Theorem 1.1. *Let a group* G *given by a presentation*

$$< x_1, \dots, x_n \mid (x_1 \dots x_n) x_j = x_j (x_1 \dots x_n), \ j \in \{1, \dots, n\}, R >,$$

where R *is a set of words on the alphabet* $X = \{x_1, \dots, x_n\}$ *such that there exists a homomorphism* $\phi : G \to \mathbb{F}_1$ *mapping each* x_j *to a generator* x *of* $\mathbb{F}_1$. *Denote by* N *the kernel of* ϕ. *If* R *is finite, then* N *is finitely presented. In particular, if* R *is empty, then* N *is a finitely generated free group.*

Proof. We have the exact sequence

$$1 \to N \to G \to \mathbb{F}_1 \to 1.$$

By the Tietze theorem,

$$\begin{aligned} G \simeq\ & < x_1, \dots, x_n, y \mid , y = x_1 \dots x_n, (x_1 \dots x_n) x_j = x_j (x_1 \dots x_n), \\ & j \in \{1, \dots, n\}, R > \simeq \\ & < x_1, \dots, x_n, y \,|\, y = x_1 \dots x_n, y x_j = x_j y, j \in \{1, \dots, n\}, R > \simeq \\ & \simeq < x_2, \dots, x_n, y \,|\, y x_j = x_j y, \ j \in \{2, \dots, n\}, \bar{R} >, \end{aligned}$$

where $\bar{R}$ is obtained from R if one changes x_1 by $y(x_2 \dots x_n)^{-1}$ in every word of R.

In terms of the last presentation, the homomorphism ϕ is organized as follows: $\phi(x_j) = x$ for each $j \in \{2, \dots, n\}$ and $\phi(y) = x^n$.

To find a finite presentation for N let us use Reidemeister – Schreier method (see, for example, §2.3 [8]). The elements x_n^k for $k \in \mathbb{Z}$ can be chosen as Schreier representatives of cosets of N in G. Then the group N is generated by

$$a_{k,j} = x_n^k x_j \overline{x_n^k x_j}^{-1} = x_n^k x_j x_n^{-(k+1)},$$

where $j \in \{2, \dots, n-1\}, k \in \mathbb{Z}$, and the elements

$$a_{k,n} = x_n^k y \overline{x_n^k y}^{-1} = x_n^k y x_n^{-(k+n)},$$

where $k \in \mathbb{Z}$. The relations

$$y x_j = x_j y, \quad j = 2, \dots, n,$$

give rise to the relations

$$a_{k,n} a_{k+n,j} a_{k+1,n}^{-1} a_{k,j}^{-1} = 1 \tag{2}$$

for $j \in \{2, \dots, n-1\}$ and $k \in \mathbb{Z}$ and the relations

$$a_{k,n} a_{k+1,n}^{-1} = 1 \tag{3}$$

for $j = n$ and $k \in \mathbb{Z}$, since

$$x_n^k (y x_j y^{-1} x_j^{-1}) x_n^{-k} =$$
$$(x_n^k y x_n^{-(k+n)})(x_n^{k+n} x_j x_n^{-(k+n+1)})(x_n^{k+n+1} y^{-1} x_n^{-(k+1)})(x_n^{k+1} x_j^{-1} x_n^{-k})$$

and

$$x_n^k (y x_n y^{-1} x_n^{-1}) x_n^{-k} = (x_n^k y x_n^{-(k+n)})(x_n^{k+n+1} y^{-1} x_n^{-(k+1)}).$$

Similarly we can rewrite the relations $\bar{R}$ and denote the result by $\tilde{R}$. Later $\tilde{R}$ will be considered in more detail and now let us show that N is finitely generated. We have

$$N = < a_{i,j}, i \in \mathbb{Z}, j \in \{2, \dots, n\} \mid (2), (3), \tilde{R} > .$$

Defining relations (2) and (3) are equivalent to

$$a_{0,n} = a_{i,n}; \tag{4}$$

$$a_{0,n} a_{i+n,j} a_{0,n}^{-1} = a_{i,j}. \tag{5}$$

where $i \in \mathbb{Z}$ and $j \in \{2, \dots, n\}$.

It follows from (5) that $a_{i+kn,j} = a_{0,n}^{-k} a_{i,j} a_{0,n}^k$ for $j \in \{2, \dots, n-1\}, i \in \{0, \dots, n-1\}$, and $k \in \mathbb{Z}$. Hence by Tietze transformations,

$$N = < a_{0,n}, a_{i,j}, i \in \{0, \dots, n-1\}, j \in \{2, \dots, n-1\} \mid \hat{R} >,$$

where $\hat{R}$ is obtained from $\tilde{R}$ by substitutions $a_{k,n} = a_{0,n}$ and $a_{i+kn,j} = a_{0,n}^{-k} a_{i,j} a_{0,n}^{k}$ in the words of $\tilde{R}$ for $i \in \{0, \dots, n-1\}, j \in \{2, \dots, n-1\}$, and $k \in \mathbb{Z}$.

So N is generated by $a_{0,n}, a_{i,j}$ for $i \in \{0, \dots, n-1\}$ and $j \in \{2, \dots, n-1\}$ the number of which is $n(n-1)+1 = (n-1)^2$. In particular, if R is empty then N is a free group freely generated by these elements.

Now let us return to $\tilde{R}$. Each $r \in \bar{R}$ gives the set of relations $\{x_n^k r x_n^{-k}, k \in \mathbb{Z}\} \subset \tilde{R}$, where $\tilde{r}_k = x_n^k \cdot r \cdot x_n^{-k}$ for each $k \in \mathbb{Z}$ is rewritten on the generators $a_{i,j}, i \in \mathbb{Z}, j \in \{2, ..., n\}$. Moreover it follows from the Reidemeister rewriting process for N that a word $\tilde{r}_l$ rewritten on $\{a_{i,j}\}$ coincides with $\tilde{r}_m$ after changing each generator $a_{l+\nu,\mu}$ by $a_{m+\nu,\mu}$ in $\tilde{r}_l$, since $a_{m+\nu,\mu} = x_n^{m-l} a_{l+\nu,\mu} x_n^{l-m}$ and $\tilde{r}_m = x_n^{m-l} \tilde{r}_l x_n^{l-m}$. Then we have from (4) and (5) that $\tilde{r}_{j+kn} = a_{0,n}^{-k} \tilde{r}_j a_{0,n}^{k}$ for each $k \in \mathbb{Z}$ and $j \in \{0, \dots, n-1\}$. Therefore $\tilde{r}_{j+kn}$ is a consequence of $\tilde{r}_j$ for each $r \in \bar{R}$, $k \in \mathbb{Z}$ and $j \in \{0, \dots, n-1\}$. So for $k \in \mathbb{Z} \setminus \{0\}$ and $j \in \{0, \dots, n-1\}$ the relations $\tilde{r}_{j+kn}$ can be removed from the set $\tilde{R}$ of defining relations of N. Since R is finite, the set $\{\tilde{r}_j = x_n^j \cdot r \cdot x_n^{-j} | j \in \{0, \dots, n-1\}, r \in R\}$ is finite. Hence N is finitely presented, i.e., Theorem 1.1 is proved.

2. Existence of non-Hopfian Hurwitz C-groups

It is well-known (see, for example, [7]) that the group

$$\tilde{G} = < a, t | t^{-1} a^2 t = a^3 >$$

is non-Hopfian. Therefore to prove part (i) of Theorem 0.6, it is sufficient to show that $\tilde{G}$ is an irreducible C-group. From the Tietze theorem we have

$$\begin{array}{rl} \tilde{G} \simeq & < a, t, x_1, x_2 \mid x_1 = t, x_2 = ta, a^2 t a^{-2} = ta > \simeq \\ & < x_1, x_2 \mid (x_1^{-1} x_2)^2 x_1 (x_1^{-1} x_2)^{-2} = x_2 > \simeq \\ & < x_1, ..., x_5 \mid x_3 = x_1^{x_2}, x_3 = x_4^{x_1}, x_5 = x_4^{x_2}, x_5 = x_2^{x_1} >, \end{array} \tag{6}$$

where $x_i^{x_j} = x_j x_i x_j^{-1}$, that is, $\tilde{G}$ is a C-group. It is an irreducible C-group, since the C-generators $x_1, \dots, x_5$ are conjugated to each other in $\tilde{G}$.

To prove part (ii) of Theorem 0.6, consider the group

$$G = \tilde{G} \times < y \mid \emptyset > .$$

We have $G/G' \simeq \mathbb{Z}^2$ and

$$\begin{array}{rl} G \simeq & < x_1, x_2, y \,|\, yx_1 = x_1y, x_2y = yx_2, (x_1^{-1}x_2)^2x_1(x_1^{-1}x_2)^{-2} = x_2 >\simeq \\ & < x_1, x_2, x_3, y \ \mid \ x_3 = (x_1x_2)^{-1}y, yx_1 = x_1y, x_2y = yx_2, \\ & (x_1^{-1}x_2)^2x_1(x_1^{-1}x_2)^{-2} = x_2 >\simeq \\ & < x_1, x_2, x_3 \ \mid \ (x_1^{-1}x_2)^2x_1(x_1^{-1}x_2)^{-2} = x_2, \\ & (x_1x_2x_3)x_i = x_i(x_1x_2x_3),\ i = 1, 2, 3 > . \end{array}$$

Therefore G is a Hurwitz C-group of degree three. Since the group $\tilde{G}$ is non-Hopfian and $G = \tilde{G}\times < y \ \mid \ \emptyset >$, the group G is also non-Hopfian.

It is easy to see that for any $k \in \mathbb{N}$ the projective Hurwitz groups $\bar{G}_{3,k} \simeq \tilde{G} \times \mathbb{Z}/3k\mathbb{Z}$ and therefore they are also non-Hopfian groups.

As is known (see, for example, [7]), non-Hopfian groups are not residually finite.

Remark. It follows from [4] that $\tilde{G}$ is also the group of a 2-knot, since the graph of the last presentation in (6) is a tree.

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